Conflicting Perspectives
On Geometrizing Spacetime

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Contents

> What is the use then of imagining an electro-tonic state of which we have no distinctly physical conception, instead of a formula of attraction which we can readily understand? I would answer, that it is a good thing to have two ways of looking at a subject, and to admit that there are two ways of looking at it.
>
> - James Clerk Maxwell,
> *On Faraday's Lines of Force* (1855–56)

1 Introduction

Geometrizing spacetime requires making several choices and these choices invite vivid disagreements. Some disagreements reveal their sources transparently, such as whether a theory should endow spacetime with local Lorentz symmetry or not [1]. Other disagreements, however, can take place between theories that resemble each other superficially, for instance, by utilizing similar mathematical tools. In such cases the conflicts may only reveal their nature and sources on further scrutiny, and the need for careful comparison becomes even more pressing when it bears on decisions concerning pursuitworthiness (either at an individual's level, as when deciding a doctoral research project, or at an institutional level, as when allocating competitive telescope time).

In this paper, we provide such a comparison between two conflicting perspectives on geometrizing spacetime that use the same mathematical objects (spacetimes possessing geometrical attributes besides curvature) to make claims that are in tension with each other. In doing so, we describe the precise source, nature and implication of this tension and, finally, provide an account of the epistemic virtues available in defense of the two competing positions.

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[1] Doubly Special Relativity [Kowalski-Glikman, 2005] and Einstein-Aether theories [Eling et al., 2005] being some well known examples of formalisms that violate Lorentz invariance.

We begin by briefly introducing the key geometrical object (metric-affine spacetimes) and the properties it admits (curvature, torsion and non-metricity). Then, we introduce the two perspectives in tension, namely, 'Metric Affine Gauge Gravity' (MAG) [Hehl et al., 1995] and 'Geometrical Trinity' (GT) [Jimenez et al., 2019] formalisms. Starting with the latter, we summarize the mathematical argument at the core of the Geometrical Trinity discourse and some of the ways these formal results have been interpreted (in particular, concerning their bearing on equivalence, empirical distinguishability, under-determination and geometric conventionalism). In contrasting the gauge gravity approach, we lead with a derivation of a simpler theory (Einstein-Cartan gravity) to show how the incompatibility arises already and generalize this tension to the more interesting case of Metric Affine Gravity.

This tension is sourced from what we call the 'Hypermomentum Challenge', whose technical content was first made explicit by Iosifidis and Hehl [2024], though the underlying tension has been latent in gauge gravity literature since Hehl et al. [1995]. In §4, we anticipate some objections and counter-objections to the hypermomentum challenge. Here, we cast the hypermomentum challenge as a trade-off between epistemic virtues and briefly relate it to wider interests in philosophy of science (such as Kuhnian and Lakatosian program comparisons).

Despite operating within overlapping theoretical territory, the two research communities have developed largely distinct bodies of literature, with rare direct engagement across the divide. Each brings its own notational conventions and community-specific vocabulary, leading to additional resistance to cross-community dialogue. One aim of this paper is to remain accessible to researchers on both sides of this disagreement and where exposition may seem familiar to one audience, it is intended as orientation for the other.

1.1 Mathematical Preliminaries

Let us begin with defining the geometric objects that are referenced frequently through this paper. The most commonly discussed tensorial object relevant to this work is the Riemann tensor that captures the curvature of a spacetime manifold, which can be written down for a general affine connection Γ as follows

$$R^{\alpha}{}_{\beta\mu\nu}(\Gamma) \equiv \partial_\mu \Gamma^{\alpha}_{\nu\beta} - \partial_\nu \Gamma^{\alpha}_{\mu\beta} + \Gamma^{\alpha}_{\mu\lambda}\Gamma^{\lambda}_{\nu\beta} - \Gamma^{\alpha}_{\nu\lambda}\Gamma^{\lambda}_{\mu\beta}. \tag{1}$$

Curvature captures a vector's deviation from itself on parallel transporting over a closed loop and is typically the geometric quantity associated with the presence of energy-momentum in spacetime theories of gravity. The next useful object to define is the covariant derivative of the metric

$$Q_{\alpha\mu\nu} \equiv \nabla_\alpha g_{\mu\nu}, \tag{2}$$

which is called non-metricity and it measures the change in a vector's length as it is parallel transported over a manifold. In metric manifolds, where there exists an invariant length element, the non-metricity tensor is zero. Now, consider a manifold where we require the curvature and non-metricity to vanish everywhere but admit an affine connection such that the bottom two indices do not commute (as opposed to the usual case of Levi-Civita connection used in GR). We denote this quantity, in a holonomic basis, as

$$T^{\alpha}_{\beta\gamma} \equiv \Gamma^{\alpha}_{\beta\gamma} - \Gamma^{\alpha}_{\gamma\beta} \tag{3}$$

where T is a bilinear map associated with the affine connections and is called the Cartan torsion tensor (or, simply, torsion). The torsion tensor $T(X, Y)$ takes vectors X and Y as its input and maps them to an output vector $T(X, Y)$ representing the displacement in the tangent space when the tangent space is rolled along an infinitesimal parallelogram with sides X and Y. Similar to how curvature is heuristically interpreted as a vectors deviation from itself when parallel transported along a closed loop, the torsion tensor can be interpreted as the non-closure of a parallelogram in that manifold.

In General Relativity, we restrict ourselves to spacetime geometries in which the straightest lines are also the distance optimizing ones [Levi-Civita, 1917, Wald, 1984]. The notion of straightness is endowed by the affine connection $\Gamma^{\alpha}_{\beta\gamma}$ and the idea of distance comes from the metric $g_{\alpha\beta}$. Roughly speaking, the 'straightness $\iff$ distance-optimizing' condition can be understood as imposing two separate mathematical restrictions:

- The affine connection is symmetric in its bottom two indices i.e. $\Gamma^{\alpha}_{\beta\gamma} - \Gamma^{\alpha}_{\gamma\beta} = 0 \implies T^{\alpha}_{\beta\gamma} = 0$. This condition imposes vanishing spacetime torsion.

- The covariant derivative of the metric, also called non-metricity, is zero i.e. $\nabla_\alpha g_{\mu\nu} = 0 \implies Q_{\alpha\mu\nu} = 0$. This is also known as the metric-compatibility condition.

In exploring alternate geometrical descriptions of gravity that still lie in a conceptual neighborhood of general relativity, one might wish to suspend these assumptions and follow up on the consequences. Or, one may simply suspend these assumptions for mathematical curiosity and investigate the larger theory-building possibilities we are afforded. The geometrical objects that violate these assumptions are called metric affine spacetime manifolds and they contain attributes, namely torsion and non-metricity, that are assumed to vanish in GR. Amongst the existing scholarship on gravitational physics with such alternatives, we find – what I will argue to be – two conflicting perspectives[2]. Crucially, the two formalisms employ the same geometrical objects for constructing their respective theories of gravity and both take as their starting point the observation that general relativity occupies only one region in a wider theory building space (as it uses only metric-compatible torsion-free manifolds). We will now discuss both of these perspectives in more detail.

2 The Geometrical Trinity of Gravitational Theories

It has been known since the early days of GR that there exists a family of theories of gravity constructed from torsion rather than curvature, usually called teleparallel theories of gravity (TGR). Historically, teleparallelism first arose in Einstein's *Fernparallelismus* programme, where distant parallelism was explored as part of a unified field theory of gravitation and electromagnetism [Sauer, 2004]. In its later revival, TGR was attractive because it was believed it could be cast as a gauge theory of spacetime translations [Cho, 1976a,b, Maluf, 2013] and because it appeared to offer a cleaner handle on questions such as the definition of gravitational energy-momentum [Møller, 1961]. However, teleparallel theories have also attracted criticism from both philosophers and physicists: the analogy with Yang-Mills gauge theory is imperfect [Hehl et al., 1995, Weatherall, 2025], the formalism appears to involve surplus structure [Weatherall and Meskhidze, 2025, Weatherall, 2025], and there are concerns regarding the mathematical coherence of teleparallel theories [Duerr and Read].

More recently, it has been shown that one may formulate a corresponding family of theories using non-metricity rather than torsion; these are known as symmetric teleparallel theories of gravity (STGR) [Nester and Yo, 1999, Adak et al., 2006]. Similar to TGR, STGR has been taken to possess several mathematically attractive features. In particular, the existence of the coincident gauge can trivialize the affine connection [Jimenez et al., 2018, 2019] and exhibit features of cosmological interest in $f(Q)$ extensions [Jiménez et al., 2020]. But STGR, too, inherits some of the same worries as TGR, including skepticism about surplus structure and about its gauge-theoretic interpretation.

For our purposes, we can ignore these branches of argumentation and, instead, focus on one particular cluster of claims concerning TGR and STGR that, hereon, we call the 'Geometrical Trinity' perspective. The first important GT claim of our interest is that there exists a theory, that is a member of the family of teleparallel theories, defined on manifolds with vanishing curvature and non-metricity, that is equivalent to General Relativity (but where matter influences, and is influenced by, spacetime torsion as opposed to curvature). This special member of the TGR family is called the Teleparallel Equivalent of General Relativity (TEGR) [Hayashi and Shirafuji, 1979, Jimenez et al., 2019]. Similarly, the second important GT claim is that there exists a theory, that is a member of the family of symmetric teleparallel theories, defined on manifolds with vanishing curvature and torsion, that is equivalent to General Relativity (but where matter influences, and is influenced by, spacetime non-metricity as opposed to curvature). *Mutatis mutandis*, we call this theory the Symmetric Teleparallel Equivalent of General Relativity (STEGR)[Nester and Yo, 1999, Jimenez et al., 2018, 2019]. Thus, from the two aforementioned claims and the transitivity of the equivalence relation, we can summarize the central proposal of Geometrical Trinity as the following — there exists a trio of equivalent theories of gravity constructed exclusively using just one of the three possible, independent geometrical attributes of spacetime i.e. GR (using curvature), TEGR (using torsion) and STEGR (using non-metricity) [Jimenez et al., 2019] [3].

[2] The term 'conflicting perspectives' (sometimes called conflicting approaches or conflicting frameworks) is an umbrella term that captures an inter-related tension in the mathematical formalisms, scientific assumptions and theoretical desiderata.

[3] One could now already anticipate the other perspective, called Metric Affine Gauge Theory of Gravity (or just metric affine gravity or gauge gravity), where we build a theory of gravity without restricting ourselves to using only

To evaluate this claim, and compare its epistemic virtues with those of the Metric Affine Gravity formalism, it is crucial to understand the precise sense in which the Geometrical Trinity claims an equivalence between the three types of theories. Clearly, the three nodes of the trinity admit different geometric intuitions and interpretations, the different geometric objects have distinct mathematical properties and the theories are not equivalent in a category theoretic sense either [Weatherall and Meskhidze, 2025]. The equivalence that is being discussed here, instead, refers to a 'dynamical equivalence' relation between the three theories. In the remainder of this section, we summarize the argument from Jimenez et al. [2019] for the dynamical equivalence across Geometrical Trinity and discuss its key implications. In the subsequent sections, we introduce the gauge gravity perspective and elaborate on the challenges it poses to the Geometrical Trinity claims.

2.1 Argument for Dynamical Equivalence

We begin with the well-known Einstein-Hilbert action defined over Riemannian spacetimes[4],

$$S_{\mathrm{GR}(2)} = \frac{1}{16\pi G}\int \mathrm{d}^4x\sqrt{-g}\,\mathcal{R}(g), \tag{4}$$

where $\mathcal{R}(g)$ is the scalar curvature computed using Levi-Civita affine connection. Varying $\mathcal{S}_{\mathrm{GR}(2)}$ with the metric yields the usual Einstein Field Equations for describing the interdependence between the derivatives of the metric field and matter dynamics. However, an alternative way of arriving at the same set of equations could be through defining an appropriate action on metric affine spacetimes (which could, in principle, be geometrically richer than Riemannian spacetimes) but using the method of Lagrange multipliers to constrain the variational calculus of optimizing the action. Then, the action for general relativity can be written as

$$S_{\mathrm{GR}_{(1)}} = \int \mathrm{d}^4x\left[\frac{\sqrt{-g}}{16\pi G}g^{\mu\nu}R_{\mu\nu}(\Gamma) + \lambda^{\mu\nu}_{\alpha}T^{\alpha}_{\mu\nu} + \hat{\lambda}^{\alpha}_{\mu\nu}Q^{\mu\nu}_{\alpha}\right], \tag{5}$$

where λ and $\hat{\lambda}$ are the Lagrange multipliers set-up to enforce vanishing torsion T and non-metricity Q respectively. Note, the curvature $R_{\mu\nu}$ in the integral for $\mathcal{S}_{\mathrm{GR}(2)}$ is defined for a general affine connection Γ (instead of $\mathcal{R}$ which uses Levi-Civita connection).

As described earlier, TEGR and STEGR also possess only one non-vanishing geometric attribute (torsion and non-metricity respectively). Consequently, we can modify the form of $\mathcal{S}_{\mathrm{GR}(2)}$ by using a scalar corresponding to the other geometrical tensors defined in Sec. 1.1 and modifying the Lagrange multipliers in order to constrain the rest as vanishing. For teleparallel gravity, the action takes the form

$$S_{\mathbb{T}} = -\int \mathrm{d}^4x\left[\frac{1}{16\pi G}\sqrt{-g}\,\mathbb{T} + \lambda_{\alpha}^{\ \beta\mu\nu}R^{\alpha}_{\ \beta\mu\nu} + \hat{\lambda}^{\alpha}_{\ \mu\nu}\nabla_{\alpha}g^{\mu\nu}\right], \tag{6}$$

where the Lagrange multipliers λ and $\hat{\lambda}$ play the role of enforcing zero curvature and non-metricity respectively. The scalar $\mathbb{T}$ is defined as

$$\mathbb{T} \equiv -\frac{c_1}{4}T_{\alpha\mu\nu}T^{\alpha\mu\nu} - \frac{c_2}{2}T_{\alpha\mu\nu}T^{\mu\alpha\nu} + c_3T_{\alpha}T^{\alpha}, \tag{7}$$

with $T_{\mu} = T^{\alpha}_{\mu\alpha}$ being the trace of the torsion tensor and c_1, c_2, c_3 as free parameters. Jimenez et al. [2019] show that if we enforce a vanishing curvature scalar $R = 0$ and restrict ourselves to metric-compatible connections, then the following relation can be derived

$$R = \mathcal{R}(g) + \mathring{\mathbb{T}} + 2\mathcal{D}_{\alpha}T^{\alpha}, \tag{8}$$

where $\mathring{\mathbb{T}}$ is just $\mathbb{T}$ after setting $c_1 = c_2 = c_3 = 1$. Notice, then, that the Ricci scalar defined for Levi-Civita connection only differs from $\mathring{\mathbb{T}}$ by a sign and a derivative term (also known as a boundary term). The boundary terms in the expression of any quantity vanish when we vary the said quantity.

one geometric object at a time. Instead, gauge gravity formalism argues that we consider spacetimes with varying degrees of all three geometrical attributes as valid solutions to field equations.

[4] Note that when we use the term 'Riemannian spacetimes', we do not refer to manifolds with positive-definite signature but, instead, follow the convention of gauge-gravity literature to refer to Lorentzian manifolds equipped with Levi-Civita connections.

Hence, the $\mathcal{D}_\alpha T^\alpha$ term does not influence the form of the field equations after optimizing the action and can be ignored while considering the derived dynamical laws. In this way, the following action

$$S_{\text{TEGR}} = -\int \mathrm{d}^4x \left[\frac{1}{16\pi G}\sqrt{-g}\,\mathring{\mathbb{T}}(g,\Lambda)\right] \tag{9}$$

for the teleparallel equivalent of GR is claimed to be dynamically equivalent to the Einstein-Hilbert action of GR. Here, we highlight a feature of $\mathcal{S}_{\text{TEGR}}$ that takes an important role later in our argument. The torsion scalar $\mathring{\mathbb{T}}$ depends independently on two degrees of freedom — first, the metric g and, second, the affine connection Γ which, for vanishing curvature and non-metricity, we parametrize using a member Λ of the general linear group $GL(4,\mathbb{R})$. We will scrutinize this feature in greater detail later in this paper.

While the case of symmetric teleparallel gravity is slightly more complicated, the general reasoning schematic remains the same. First, we write action for metric-affine spacetimes with scalar involving non-metricity

$$S_{\mathbb{Q}} = -\int \mathrm{d}^4x \left[\frac{1}{16\pi G}\sqrt{-g}\,\mathbb{Q} + \lambda_\alpha{}^{\beta\mu\nu}R^\alpha{}_{\beta\mu\nu} + \lambda_\alpha{}^{\mu\nu}T^\alpha{}_{\mu\nu}\right] \tag{10}$$

where the non-metricity scalar is defined

$$\mathbb{Q} = \frac{c_1}{4}Q_{\alpha\beta\gamma}Q^{\alpha\beta\gamma} - \frac{c_2}{2}Q_{\alpha\beta\gamma}Q^{\beta\alpha\gamma} - \frac{c_3}{4}Q_\alpha Q^\alpha + (c_4-1)\tilde{Q}_\alpha\tilde{Q}^\alpha + \frac{c_5}{2}Q_\alpha\tilde{Q}^\alpha, \tag{11}$$

and $Q_\alpha = Q^\lambda_{\alpha\lambda}$ and $\tilde{Q}_\alpha = Q^\lambda_{\lambda\alpha}$ are the two independent traces of the non-metricity tensor, c_i corresponds to five free parameters and λ represents the Lagrange multipliers tuned to enforce vanishing curvature and torsion. Again, Jimenez et al. [2019] show that for a torsion-free connection, the Ricci scalar can be related to the non-metricity scalar as

$$R = \mathcal{R}(g) + \mathring{\mathbb{Q}} + D_\alpha\left(Q^\alpha - \tilde{Q}^\alpha\right), \tag{12}$$

with $\mathring{\mathbb{Q}}$ is just $\mathbb{Q}$ with $c_i = 1$ for $i = \{1...5\}$. As before, in case $R = 0$, we can ignore the boundary term and the action is reduced to

$$S_{\text{STEGR}} = -\frac{1}{16\pi G}\int \mathrm{d}^4x\sqrt{-g}\,\mathring{Q}(g,\xi), \tag{13}$$

which is claimed to be dynamically equivalent to $\mathcal{S}_{\text{GR(2)}}$ and corresponds to the theory named Symmetric Teleparallel Equivalent of General Relativity. Notice, again, $\mathring{\mathbb{Q}}$ has two independent degrees of freedom i.e. the metric tensor g and the affine connection Γ, which in this case (of vanishing torsion) is parametrized by a set of functions ξ^α.

2.2 Interpreting Dynamical Equivalence

It is important to emphasize that actions that differ by boundary terms can yield theories which are different in several non-trivial and interesting ways. For instance, it is well-known that using the Einstein-Hilbert action does not lead to a well-posed variational problem. Thus, to derive general relativity, one must include the Gibbons-Hawking-York term [Gibbons and Hawking, 1977, York, 1972], which has an important relationship with blackhole entropy. One does not require such an additional term for varying $\mathcal{S}_{\text{TEGR}}$ or $\mathcal{S}_{\text{STEGR}}$ because they only contain at most a first derivative of the metric. Additionally, the trinity theories can have different properties concerning the asymptotic structure of spacetime. In general relativity, several important results are derived by defining quantities at infinity [Geroch, 1977] (more specifically, on a region of a conformal completion of spacetime possessing appropriate geometrical properties such as smoothness, topology, etc.). Thus, results concerning asymptotic symmetry groups, gravitational waves in full non-linear theory, positivity of Arnowitt-Deser-Misner mass, etc. would require explicit and focused analysis for checking which insights can be exported out of GR onto the other nodes of Geometrical Trinity. Relatedly, the Newtonian limits of TEGR and STEGR have been derived and shown to be mutually equivalent [Wolf et al., 2024a], providing a useful consistency check for the GT formalism. However, the more structurally revealing analysis using Ehlers' frame theory [Ehlers, 2019] (which exposed the degenerate structure of the

Newtonian limit of GR) has not yet been carried out for the trinity theories. Whether this analysis would uncover analogous features in the teleparallel or symmetric teleparallel settings remains an open question.

Irrespective of these differences, Wolf et al. [2024b] claim that this dynamical equivalence across Geometrical Trinity theories is conceptually interesting for the following reason — if the actions that differ by boundary term yield equivalent field equations, then the particle trajectories in all three of these theories coincide (barring some subtlety concerning coupling between metric and matter fields, a detailed discussion of which we defer until next section). Thus, insofar as our probes of the spacetime geometry rely entirely on the paths of particles traced in spacetime, the three Geometrical Trinity theories remain indistinguishable. We emphasize the implication of this claim (because, in the next section, we will attempt to deflate it) that the Geometrical Trinity theories are not merely empirically indistinguishable (which would be an equivalence contingent on our current observational prowess and possibly broken in the future with access to better data). Instead, the Geometrical Trinity position is that no amount of experimental refinements involving increasingly precise measurements of particle trajectories would ever be able to differentiate the solutions of field equations in GR from their corresponding equivalents in TEGR or STEGR. While the assumption that the only probes of spacetime geometry are particle trajectories is contentious if unqualified[5], for the purposes of this paper, we shall assume it to be true. This is because the challenge from gauge gravity formalism that we discuss later is already incompatible with Geometrical Trinity's dynamical equivalence claim, even if we restrict ourselves to observing particle trajectories.

We make one last comment in this subsection before moving towards discussing the gauge gravity formalism. There are various discussions in philosophy of physics literature concerning whether the dynamical equivalence of Geometrical Trinity leads to an under-determination in assessing statements about spacetime geometry[6] [Knox, 2011, Mulder and Read, 2024, Wolf et al., 2024b]. In particular, Dürr and Read [2024] use this GT under-determination to constrain what ontological commitments remain palatable (notable removals from the list being realism about spacetime geometry, realism about spacetime curvature, etc.) and, also, to reinforce support for Geometric Conventionalism (i.e the view, in the tradition of Poincaré and Reichenbach, that the choice between empirically equivalent geometric descriptions of spacetime is conventional rather than factual). It is worth clarifying, that in describing the tensions between Geometrical Trinity and gauge gravity claims, we do not attempt to undermine geometric conventionalism or, more broadly, make physics-informed-metaphysics claims[7]. Instead, we frame this conflict in perspectives on geometrizing spacetime as a philosophically interesting case study of a more general trade-off between epistemic virtues. Before doing that, though, we motivate and summarize some relevant aspects of the second formalism of our interest - Metric Affine Gauge Gravity.

3 Metric Affine Gauge Theory of Gravity

There is a long and rich history of trying to bridge the study of gauge symmetries with the study of spacetime geometry going back to the pioneering work by Weyl [1929], Utiyama [1956], Kibble [1961], Sciama [1962]. The brief introduction in this section would aggressively condense the exposition into a minimal conceptual core required for our argument. We begin by noting that, similar to how matter's motion is represented by an evolution of its external degrees of freedom, changes in the states of matter can be represented by an evolution of its internal degrees of freedom. Standard examples of

[5] Wolf et al. [2024b] indeed discuss gravitational waves as modern tests of GR. However, in absence of a full non-linear analysis of the Geometrical Trinity theories at their null infinities, it remains difficult to make concrete claims about the nature of expected waveforms.

[6] It is worth noting that GT proponents themselves acknowledge that the dynamical equivalence does not survive the passage to modified gravity theories. At the level of $f(Q)$, $f(R)$, and $f(T)$ extensions, the degeneracy is broken [Heisenberg, 2024]. GT researchers suggest this as an empirical opening claiming that cosmological observations can constrain the choice between these modified theories. Although interesting both in their own right and in how they influence the understanding of underdetermination, these higher-order extensions of the gravitational Lagrangian do not have decisive observational support and do not directly concern the argument presented in this paper.

[7] On the question of physics-informed-metaphysics, we cite the wonderfully lucid Howard Stein — 'To borrow from the ancient philosophical tradition, what I believe the history of science has shown is that on a certain very deep question Aristotle was entirely wrong, and Plato — at least on one reading, the one I prefer — remarkably right: namely, our science comes closest to comprehending "the real", not in its account of "substances" and their kinds, but in its account of the "Forms" which phenomena "imitate" (for "Forms" read "theoretical structures", for "imitate", "are represented by").' [Stein, 1989]

such internal structure are quantities like electric charge, spin, and other properties that characterize *what* matter is rather than *where*. General relativity serves as a remarkable theory for understanding the evolution in external dimensions by describing it as a consequence of the non-linear relationship between spacetime geometry and matter components. Similarly, the evolution in internal degrees of freedom are very well described by a family of gauge theories that are built from considerations of local, non-rigid symmetry transformations of the target system. The pinnacle of theory building using this so-called gauge procedure is the Standard Model of Particle Physics, the predictions of which remain some of the most precisely verified claims in all science [Fan et al., 2023].

There are two possible approaches to the problem of bridging our mathematical understanding of evolution in internal and external dimensions. One possibility is to try to elevate the internal dimensions to the same status as the external ones by subjecting them to similar mathematical treatment (the most well-known consequence of which are the extra dimensions in Kaluza-Klein like theories [Kaluza, 1921, Klein, 1926]). Alternatively, one could try to re-construct a theory resembling general relativity but through the procedure employed in the construction of gauge theories. The later approach is what we will focus on in this section and, as we will show, it naturally motivates a generalization from using Riemannian spacetimes to using metric-affine spacetimes.

The assembly of a gauge theory of gravity can be presented in the following steps of the aforementioned gauge procedure that extracts dynamical laws from a given symmetry group. The first step is to identify the appropriate symmetry group to feed into the gauge procedure. The symmetry group most commonly associated with the special-ness of general relativity is the diffeomorphism group Diff(M). However, there are known reasons for why Diff(M) is unsuitable for accomplishing the goal of constructing a gauge theory of gravity. From a top-level, the reasons can be summarized in the following observations. Unlike the groups corresponding to the gauge theories in particle physics, diffeomorphism group is infinite dimensional. This leads to difficulties in finding the representation theory of the group that is required to discuss source fields in the theory [Hehl et al., 1995, Hehl, 2014]. Further, Diff(M) is not an automorphism of any principle-fiber bundle [Trautman, 1980, Gomes, 2022]. This means that, unlike other gauge theories, elements of the diffeomorphism group also act on the base manifold M (instead of only affecting the fibers of a principle bundle $P \to M$ defined over the fixed base manifold M).

The Poincaré group $\mathrm{ISO}(1,3) = \mathbb{R}^{1,3} \rtimes SO(1,3)$, composed of spacetime translations P^a (where the unorthodox notation is to denote torsion with T^α later) and Lorentz transformations M^{ab}, is the next most natural symmetry group appropriate for producing a theory of gravity using the gauge procedure [Kibble, 1961, Sciama, 1962]. The physical motivation comes from Noether's theorem that demonstrates that invariance under spacetime translations and Lorentz transformation generates, respectively, the conserved stress-energy tensor $T^{\mu\nu}$ and the conserved angular momentum tensor $S^{\mu\nu\rho}$ [Hehl et al., 1976c]. These are precisely the currents that characterize the matter content of any relativistic field theory and, thus, the symmetry group whose conserved charges are carried by matter is the natural candidate for constructing a theory of gravity. In what follows, we build a minimal gauge theory of gravity arising from Poincare group (namely, Einstein-Cartan theory) as it already contains the essential challenge to the Geometrical Trinity. Later, we will describe how the argument extends to full metric-affine gauge gravity setting and puts pressure on all three nodes of the trinity.

The gauge procedure associates a potential to each generator of the symmetry group. Corresponding to the translation generators P^a, we obtain a set of one-form fields $\vartheta^a = e^a{}_\mu dx^\mu$, called the co-frame (or vierbein). Corresponding to the Lorentz generators M^{ab}, we obtain the connection one-forms $\Gamma^a{}_b = \Gamma^a{}_{b\mu} dx^\mu$, called the affinity. These two objects are the fundamental dynamical variables of the theory, playing the role that the metric $g_{\mu\nu}$ and the connection coefficients $\Gamma^\alpha_{\mu\nu}$ play in the standard formulation of GR. To make contact with the index notation used in the preceding sections, we note that the metric is recovered from the co-frame as

$$g_{\mu\nu} = \eta_{ab}\, e^a{}_\mu\, e^b{}_\nu, \tag{14}$$

where η_{ab} is the Minkowski metric, and the affinity $\Gamma^a{}_{b\mu}$ encodes the same information as the connection coefficients appearing in Eq. (1), with one index projected into the local inertial frame.

From the gauge potentials, we construct the field strengths by taking their exterior derivatives[8]

[8]Exterior derivatives d are simply the generalization of derivative operator to one-forms, such that $dA = \partial_\mu A_\nu dx^\mu \wedge dx^\nu$.

and adding the appropriate wedge product[9] terms (to account for the non-commutativity of Poincare group transformations). Corresponding to the co-frame and affinity respectively, the field strengths are the torsion and the curvature

$$T^\alpha \equiv d\vartheta^\alpha + \Gamma^\alpha{}_\beta \wedge \vartheta^\beta = \frac{1}{2} T^\alpha{}_{ij}\, dx^i \wedge dx^j, \qquad R_\alpha{}^\beta \equiv d\Gamma_\alpha{}^\beta - \Gamma_\alpha{}^\gamma \wedge \Gamma_\gamma{}^\beta = \frac{1}{2} R_{ij\alpha}{}^\beta\, dx^i \wedge dx^j, \tag{15}$$

which the reader will recognize as the torsion tensor $T^\alpha{}_{\mu\nu}$ and Riemann tensor $R^\alpha{}_{\beta\mu\nu}$ from Sec. 2.1, now expressed as differential forms. Having identified certain 2-forms as the field strengths of the gauge potentials, we proceed to the final step of assembling and varying an action for the dynamics of the field strength. The geometrical constraint on the action is that it needs to be a 4-form, so that it can be integrated over a 4D manifold. One possibility then is to construct a 4-form by wedging the field strength 2-form $R_{\alpha\beta}$ with so-called volume form[10] $\eta_{\alpha\beta} \equiv \frac{1}{2}\epsilon_{\alpha\beta\gamma\delta}\, \vartheta^\gamma \wedge \vartheta^\delta$ (i.e. the simplest 2-form constructed by wedging two co-frames). Then, ignoring the cosmological constant term, the action assembled becomes

$$S_{\rm EC} = \frac{1}{2\kappa} \int_M \left[R^{\alpha\beta} \wedge \eta_{\alpha\beta} \right] + S_m \tag{16}$$

where, unlike when we discussed the actions of TEGR and STEGR, we have included matter action term S_m explicitly in the expression. In general, reflecting the dynamical degrees of freedom of the geometric part of the action, the matter Lagrangian $\mathcal{L}_m$ can depend on both gauge potentials such as

$$S_m = S_m(\vartheta, \Gamma) \equiv \int \mathcal{L}_m\,(\vartheta, \Psi, D\Psi) \tag{17}$$

where Ψ is the matter field and the affinity couples to matter action through the covariant derivative of the matter field $D\Psi = d\Psi + \Gamma\Psi$. The EC in the subscript of Eq. (16) refers to Einstein-Cartan action (or Einstein-Cartan-Sciama-Kibble action). Notice, until this point, the Einstein-Cartan action differs structurally from the Einstein-Hilbert action only in the explicit inclusion of the matter term S_m (and in that it is not defined on a Riemannian spacetime but over a Riemann-Cartan spacetime). So, the geometric sector of the actions being discussed in the Geometrical Trinity and gauge gravity formalisms is the same.

We will finally address the issue of two dynamical degrees of freedom which we had flagged in the previous subsection. Within the gauge gravity literature (and wider physics in general), unless discussed explicitly, one would derive the field equations by varying the action with all of its degrees of freedom to arrive at the complete set of the coupled differential equations describing the field dynamics. On varying the action with the co-frame, we get a set of equations [Hehl, 2014],

$$\mathrm{Ric}_\alpha{}^i - \frac{1}{2} e^i_\alpha \mathrm{Ric}_\gamma{}^\gamma = \frac{\kappa}{e} \mathcal{T}_\alpha{}^i\,, \tag{18}$$

that relate Ricci curvature tensor computed for the full Riemann-Cartan connection $\mathrm{Ric}\,(\vartheta, \Gamma)$ to the canonical energy-momentum tensor of matter $\mathcal{T}$ (serving as the analog of Einstein-Field equations for Riemann-Cartan geometry, expressed in terms of the vierbein formalism). These equations, however, are not a complete description of the system's dynamics because for that we must also vary the action with affinity. On doing so, one arrives at the following equations [Hehl, 2014]

$$\mathrm{Tor}_{\alpha\beta}{}^i + e^i_\alpha\, \mathrm{Tor}_{\beta\gamma}{}^\gamma - e^i_\beta\, \mathrm{Tor}_{\alpha\gamma}{}^\gamma = \frac{\kappa}{e}\, S_{\alpha\beta}{}^i, \tag{19}$$

that relate the torsion of spacetime Tor with the spin angular momentum density of matter $S^i_{\alpha\beta} \equiv \delta\mathcal{L}_m / \delta\, \Gamma^{\alpha\beta}_i$. For a scalar field, the spin angular momentum density is always vanishing but it can be non-zero in the case of Dirac spinors (which are useful to model particles like electrons and quarks but the relationship of which with spacetime torsion is yet to be experimentally probed). This second class of field equations, concerning affine connection and intrinsic spin of matter, are neither present in the Riemannian setting of GR nor discussed in the context of Geometrical Trinity results. Here arises the incompatibility between the claims of gauge gravity and Geometrical Trinity. We will now

[9] A wedge product is an anti-symmetric map that takes two differential forms and produces a higher-order form $dx^i \wedge dx^j = -dx^j \wedge dx^i$

[10] Note that the volume form $\eta_{\alpha\beta}$ written with greek indices is distinct from the Minkowski metric η_{ab} written with roman ones.

explore the nature and implications of this incompatibility, extend this comparison to a gauge gravity theory more general than Einstein-Cartan (i.e. Metric Affine Gravity) and evaluate what epistemic warrants can one appeal to in defense of these two stances.

3.1 The Upshot and The Challenge

The two field equations derived above admit a compact summary: in the Riemann-Cartan formulation of gravity, energy-momentum sources curvature and spin angular momentum sources torsion. The first relationship is the familiar one from GR; the second is an additional dynamical ingredient in our physical descriptions. It implies that matter with non-trivial internal structure (specifically, matter carrying a non-vanishing spin angular momentum density ${S_{\alpha\beta}}^i$) couples differently to spacetime geometry than structureless matter. In a torsionful spacetime, the trajectories of particles with intrinsic spin deviate from those in a torsion-free spacetime, even when the curvature is identical or vanishing [Iosifidis and Hehl, 2024].

Juxtaposing this with the Geometrical Trinity claims makes the tension immediate. GT asserts an underdetermination of spacetime geometry on the basis that particle trajectories are degenerate across configurations of curvature, torsion, and non-metricity, rendering the three nodes of the trinity in-principle indistinguishable. The gauge gravity formalism rejects this directly because for matter with non-vanishing spin angular momentum, there is no dynamical equivalence on which the underdetermination can be grounded. From the gauge gravity perspective, GT's equivalence claim implicitly assumes ${S_{\alpha\beta}}^i = 0$, an assumption MAG holds to be unmotivated within the gauge-theoretic framing.

Granting validity to the gauge procedure, thus, deflates GT's position from a claim of in-principle indistinguishability to one of contingent empirical indistinguishability. The detection of spacetime torsion, presently, faces compounding observational difficulties. Gravity is intrinsically a very weak force, and unlike mass-energy, intrinsic spin tends to cancel when aggregated into macroscopic bodies (and thus observations from Gravity Probe B [Mao et al., 2007] do not track the spin-torsion coupling relevant to our discussion). Yet, proposals exist for detecting torsion signatures [Hehl et al., 2013, Puetzfeld and Obukhov, 2014] using both table-top experiments and astrophysical observations and, while the experimental program remains nascent, there are already dedicated conferences for the field[11,12].

The challenge extends as one generalizes the gauge group. One can write a more general form of the EC action by including the terms quadratic in torsion as well and arrive at the Poincare Gauge Theory[13] [Hehl et al., 1976c], which is also a curvature- and torsion-full description of gravity with identical matter coupling (but now compatible with torsion propagating through vacuum spacetimes). Furthermore, we can replace ISO(1,3) with the full affine group $GA(4,\mathbb{R}) = \mathbb{R}^4 \rtimes GL(4,\mathbb{R})$ such that it liberates the connection from metric-compatibility, allowing non-metricity alongside torsion in the manifold. The sourcing current generalizes to the full hypermomentum tensor ${\Delta^\lambda}_{\mu\nu} \equiv -e\frac{\delta \mathcal{L}_m}{\delta {\Gamma^\lambda}_{\mu\nu}}$, decomposing into spin, dilation, and shear under $GL(4,\mathbb{R})$ [Hehl et al., 1995]. Now, STEGR also loses its equivalence claim because matter with non-vanishing intrinsic dilation or shear sources non-metricity. The GT under-determination, already challenged at the Einstein-Cartan level, is fully undermined in the metric-affine setting if hypermomentum is not assumed to be vanishing. Since hypermomentum is a lesser known object in the philosophy of physics literature, and since spin-angular momentum is simply the anti-symmetric part of hypermomentum, it warrants a brief description. As we will describe later, evaluating the epistemic virtues of GT and MAG is essentially an exercise in contrasting different attitudes towards the hypermomentum tensor.

[11] See: TORSION 2026 — Tests Of Relativity with Spin InteractiONs, www.nucleares.unam.mx/torsion

[12] The deflation of the claim from dynamical to empirical equivalence bears on assessing the pursuit-worthiness of this field. For instance, someone convinced of an under-determination between curved and twisted spacetimes may not find pulsar-based searches for spacetime torsion productive telescope time. On the other hand, those skeptical of the GT may wish to proceed to build such experiments regardless.

[13] The action of the Poincare Gauge Theory can be expressed as

$$S_{\mathrm{PGT}} = \frac{1}{2\kappa}\int_M \left[R^{\alpha\beta} \wedge \eta_{\alpha\beta} + \sum_{I=1}^{3} a_I \, {}^{(I)}T^\alpha \wedge \star {}^{(I)}T_\alpha \right] + S_m \tag{20}$$

where the wedge product terms with the Hodge dual represent ${}^{(I)}T^\alpha \wedge \star {}^{(I)}T_\alpha$ the three irreducible components of torsion under $SO(1,3)$ and a^I are the coupling components. Einstein-Cartan theory can be recovered by setting $a_I = 0$

3.2 On Hypermomentum

The energy-momentum tensor $T_{\mu\nu}$ has received sustained philosophical attention in the past. Considerations of its energy conditions, localizability and coupling-procedures have generated substantial discourse amongst physicists and philosophers (for examples, see Hoefer [2000], Lehmkuhl [2011], Curiel [2014], Ferreiro et al. [2025]). The applicability of conceptual accounts one could give for energy-momentum (e.g. Curiel's interpretation of $T_{\mu\nu}$ as a concomitant of the metric in the language of jet-bundles [Curiel, 2012]) has not yet been mapped onto the hypermomentum tensor $\Delta^{\lambda}_{\mu\nu}$. Nevertheless, investigations of hypermomentum have carved out a certain niche amongst people working on gauge gravity. Here, we summarize some results that can provide a foothold for future philosophical inquiry.

A heuristic route by which hypermomentum has been motivated in the gauge-gravity literature comes from a recurring analogy, due especially to Hehl and collaborators [Hehl et al., 1976a,b], with a generalization of continuum mechanics. In the theory of polar continuum media, one extends ordinary elasticity theory by allowing the microscopic constituents of a medium to possess internal degrees of freedom, including independent rotations and more general deformations. The associated response currents decompose into parts corresponding, again, to spin, dilation, and shear. Historically, this analogy reaches back to the Cosserat brothers's work on generalized elasticity [Cosserat and Cosserat, 1909] that Cartan explicitly took as inspiration for his work on torsion a decade later [Cartan, 1922, Shapiro, 2002].

Indeed, in the literature surrounding Einstein–Cartan theory, and later metric-affine gravity, the analogy between microscopic degrees of freedom of spacetime geometry and polar continuum media is often taken very seriously. For strong supporters of relationalist interpretations of spacetime, this might already be problematic because it seems to encourage a substantival reading of spacetime as a kind of underlying material bearer of properties. Hehl, for instance, does not find this problematic because in his interpretation the existence of $T_{\mu\nu}$ already entails a form of substantivalism[14]. However, we emphasize that this is only one potential account of hyper-momentum and our observations regarding the tension between GT and MAG are agnostic towards the chosen interpretation. One could even adopt principled silence over questions of ontology and simply treat the tensor as the object that is naturally obtained when applying the gauge procedure to the general affine group (similar to how spin angular momentum naturally comes from constructing a gauge theory using the Poincaré group). The mathematical status of hypermomentum within metric-affine or Poincaré-gauge frameworks is perfectly clear: it is the source conjugate to the affine connection. In particular, if one compares these frameworks with teleparallel or symmetric teleparallel approaches, the Riemann–Cartan and Metric-Affine setting appears comparatively robust. As Jim Weatherall puts it, "Geometrically, the Riemann-Cartan geometry on which PGT is based is unimpeachable, and I am not aware of any criticisms that theory is mathematically problematic" [Weatherall, 2025].

Another interesting aspect of hypermomentum that has been explored in metric affine literature is the study of fluid solutions for the so-called *hyperfluids* [Obukhov and Tresguerres, 1993, Iosifidis, 2021b]. Beginning with work by Obukhov and Tresguerres and subsequently extended in later treatments, there has been an interest in developing cosmological solutions that feature non-vanishing hypermomentum and are governed by the dynamics of metric-affine gravity [Iosifidis, 2021a]. Within Friedmann–Lemaître–Robertson–Walker symmetry, such models source specific combinations of torsion and non-metricity and thereby modify the effective cosmological dynamics. More recent work has derived generalized Friedmann equations for quadratic metric-affine theories in the presence of cosmological hyperfluids [Iosifidis et al., 2023] and even begun to attempts to make contact with observational data [Chaudhary and Hussain, 2025]. What remains much less developed, at least relative to the mature cosmological phenomenology of more familiar modified-gravity frameworks, is a systematic bridge from these homogeneous solutions to nonlinear structure formation and other precision probes of late-time cosmology. We mention this to illustrate that, even though many open questions remain about the empirical accessibility of hypermomentum, it is still a sufficiently well-defined quantity that can facilitate potential observables in the future.

[14] Mentioned in personal correspondence

4 Assessing Epistemic Virtues

Having discussed the technical details that generate the tensions between Geometrical Trinity and Gauge Gravity formalism, we highlight the more general epistemic lesson that this case study represents. To do so, we must anticipate the objection and counter-objections that Geometrical Trinity proponents could raise against the hypermomentum related challenge.

We begin by restating the tension that we began tracing — GT claims theories of spacetime based on torsion, curvature and non-metricity are dynamically equivalent whereas no such equivalence exists as per gauge gravity formalism. We followed this tension to the differing attitudes towards assumptions on vanishing or non-vanishing of hypermomentum.

Given what was summarized about the empirical status of hypermomentum, the most natural defense for GT could be an appeal to parsimony and a charge against metric affine gravity of speculatively proliferating a new field to break the equivalence. In a stronger version of this defense, one can claim that hypermomentum does not even exist as an object in the GT theories by virtue of the Lagrange multiplier choices and, hence, its vanishing should not be considered as matter-sector assumption of the theory. If one assumes that this defense passes, then the burden of proof effectively shifts to gauge gravity researchers to show intrinsic spin couples to torsion. There are certain issues, however, that resist this shifting in the burden of proof.

The first objection against the parsimony of the trinity theories is that the hypermomentum object exists already within the GT formalism. It is standard practice to vary an action with all degrees-of-freedom and to assume one sector of the dynamical structure as trivial is the speculative excess that is not derivable from within the theory. One could respond, however, that this is a non-issue for GT if it concedes that they are not trying to build a gauge theory of gravity[15] and, for the purposes of their argument, they are not beholden to mimicking the standard implementation of the gauge procedure. Then, too, gauge gravity theorists can push-back against the speculative excess charge by claiming a parsimony in the principles of theory-building. Implementing the gauge principle in constructing both particle physics theories and gravitational physics theories enhances the inter-theoretic coherence between the two by grounding them both in the same fundamental procedure.

It is worth explicitly noting that this discussion has considered three distinct (albeit inter-related) forms of parsimony — ontological parsimony (positing fewer fundamental constituents in physical theories), epistemic parsimony (positing fewer assumptions lacking epistemic warrants) and methodological parsimony (positing fewer physical 'sectors' requiring independent theory-building procedures). Within history of physics, there are various instances of research programs that trade one flavor of parsimony to gain a form of another. In one reading, the early support for GR itself can be claimed as a form of ontological inflation (positing curvature as a new property of spacetime) in order to implement the equivalence and relativity principles (which famously deflate the assumption that there exists a class of privileged observers). This can be cited as historical precedence for mathematical sophistication or a form of ontological inflation at a cost of implementing a principle that achieves epistemic or methodological deflation.

Yet, one could remain skeptical of the the fundamental desideratum of Riemann–Cartan and Metric–Affine Gravity to cast GR as a Yang-Mills like gauge theory. For instance, one can cite fundamental differences between internal and external degrees-of-freedom (and their corresponding mathematical structures) to highlight why the gauging gravity is an unjustified endeavor. **¡Summarize or cite Hehl's defense for gauging gravity and extending to Affine group¿**An even stronger skeptic may raise concerns about utilizing the gauge procedure as a legitimate theory building device, irrespective of its past successes, citing issues with over-reliance on symmetry arguments given the absence of an account of why they should work in the first place.**¡summarize or cite Henrique (from his Cambridge elements) and other defenses for the gauge procedure¿**. Summarizing and evaluating the status of these debates falls beyond the scope of this paper.

There are proposals for performing theory choice with competing epistemic virtues. For instance, Kuhnians might suggest evaluating the virtues other than just accuracy and consistency such as scope, simplicity and fruitfulness [Kuhn, 1977]. That metric-affine gauge gravity possesses the tools to discuss matter's microstructure and potentially generate novel empirical contacts, albeit at the expense of mathematical sophistication, counts in its favor on the grounds of scope and fruitfulness.

[15] Two important issues with this retreat already arise. First, Jimenez et al. [2019] explicitly accept TGR as a gauge theory and, hence, cannot discard the gauge approach *a priori*. Second, there are strong unresolved objections against the gauge status of TGR or STGR.

Yet, one could still disagree how these virtues ought to be weighed against each other. Alternatively, a Lakatosian comparison [Lakatos, 1978] might entail identifying the two frameworks as either degenerating or progressing (i.e. whether they succeed or fail in predicting novel empirical facts). There are, however, two well-known shortcomings of Lakatosian program comparisons. The categories are neither mutually exclusive nor mutually exhaustive. These shortcomings become progressively severe in long periods of absent empirical input, whereby the two categories blur into each other. In the present case, both programs can be made to appear progressive or degenerating depending on how one interprets their treatment of matter coupling. The Geometrical Trinity can be seen as degenerating if the restriction to vanishing hypermomentum functions only to preserve equivalence without yielding new predictions. Conversely, Metric-Affine Gravity risks degeneration if the status of hypermomentum coupling continues to remain experimentally inaccessible. Others have argued [Laudan, 1977, Branahl, 2024] that Lakatosian comparisons are unsuitable for parsing the full complexity of modern research landscape and, here too, they cannot definitively arbitrate the MAG-GT disagreement.

A strong reading of the account presented in this section could be that if one is generally sympathetic to gauge theories and finds the demand to gauge gravity acceptable, then the claims of equivalence by Geometrical Trinity researchers are unconvincing because they depend on matter sector assumptions that are left implicit and appear ad-hoc. The more modest reading of the account here, which we feel more comfortable endorsing, is that – 1) a practicing physicist makes trade-offs between various epistemic virtues in evaluating pursuit-worthiness of a program that are better left acknowledged than implicit or unexamined; and 2) the incompatibility between Geometrical Trinity and Gauge Gravity claims in the literature is one case study of this general trade-off.

5 Conclusion

The comparison developed in this paper reveals a layered form of underdetermination. At the first level, the Geometrical Trinity asserts an underdetermination between curvature, torsion, and nonmetricity by an appeal to dynamical equivalence. At a second level, the gauge gravity perspective challenges this claim by exhibiting matter couplings (through spin and, more generally, hypermomentum) for which no such equivalence obtains. Given the unclear observational status of hypermomentum, this results in a meta-underdetermination: a disagreement over whether the original underdetermination is physically salient or an artifact of restrictive assumptions on the matter sector. We argue that this is the philosophically interesting locus of the MAG-GT conflict. While GT claims that the different geometrical objects can be reduced into representation of the same real world phenomena, gauge gravity approaches consider it more fruitful to associate different features of the world to distinct mathematical objects. Thus, it is a disagreement about where to place the speculative burden between the side that introduces hypermomentum as a novel physical current or on side that sets it to zero.

We have not claimed that the gauge procedure uniquely mandates any particular interpretation of hypermomentum. Still less have we taken a position on geometric conventionalism broadly construed or attempted to derive any conclusions over spacetime ontology. Instead, the more modest claim is that the incompatibility between GT and MAG reflects a general tension in the epistemic desiderata from the theory-building practice. Crucially, this tension bears on decisions about which research programs deserve sustained investment.

The question of which framework holds up under further scrutiny could be cast as one that has an empirical bearing, though any meaningful probes remain outside our current capabilities. If hypermomentum couplings admit observational contact, the meta-underdetermination resolves in favor of the gauge gravity perspective. If they remain inaccessible, the parsimony arguments for GT become correspondingly stronger. We hope to have made the nature and stakes of that choice clearer through this work.

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