Gravity in Undergraduate Thermal Physics Courses

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Abstract

A pedagogical aid is proposed for undergraduate thermal physics courses to introduce students to how the inclusion of gravity challenges the conventional formulations of the laws of thermodynamics. The aim is to stimulate deeper interest in thermal physics by revealing its conceptual overlap with general relativity—an intersection often overlooked in standard curricula.

1 Introduction

From cosmology to information theory, thermal physics has quietly shaped some of the most profound developments in modern science. Yet one of its most surprising intersections—that between thermodynamics and gravity—remains largely inaccessible to undergraduate students, obscured by the technical prerequisites of general relativity.

This paper proposes a pedagogical aid that can be incorporated into standard undergraduate thermal physics courses shortly after the introduction of the second law. Building on a reformulation by Santiago and Visser [2], which casts the Tolman–Ehrenfest effect in the language of special relativity, this framework opens a window into deep conceptual terrain without requiring a formal background in general relativity.

I present three interconnected arguments that invite students to rethink thermal equilibrium in the presence of gravity. The first is a classical argument, attributed to Maxwell, which shows that temperature gradients at equilibrium lead to a violation of the second law. The second argument, from Santiago and Visser, demonstrates that such gradients must in fact exist in gravitational fields—a result consistent with relativistic effects, not classical intuitions. The third considers whether electric fields might also induce equilibrium temperature gradients, ultimately revealing that while electromagnetism is not universal, its effects can influence equilibrium indirectly—through gravity itself.

2 Maxwell's Argument

Maxwell argued, on purely physical grounds, that temperature gradients cannot exist within bodies at thermal equilibrium [1].

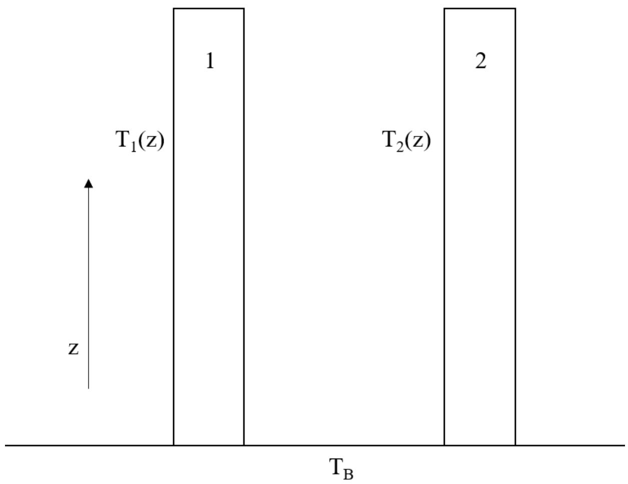

Figure 1: The hypothetical two column setting for Maxwell's argument

Imagine two vertical columns placed atop a thermally conducting surface. The base of each column is in thermal contact with this surface, ensuring equilibrium at $z = 0$ (see Fig. 1). Now suppose, hypothetically, that despite the system being in thermal equilibrium, both columns exhibit temperature gradients along the z-axis.

If, at any height z, we find that $T_1(z) > T_2(z)$ (or vice versa—the labeling is arbitrary), we could insert a horizontal conducting rod between the two columns at that level. Heat would then flow from the hotter to the cooler column. Part of this heat would descend through column 2 to its base, and from there conduct laterally through the shared surface, eventually heating the base of column 1. That in turn drives heat upward through column 1—completing a cycle (see Fig. 2).

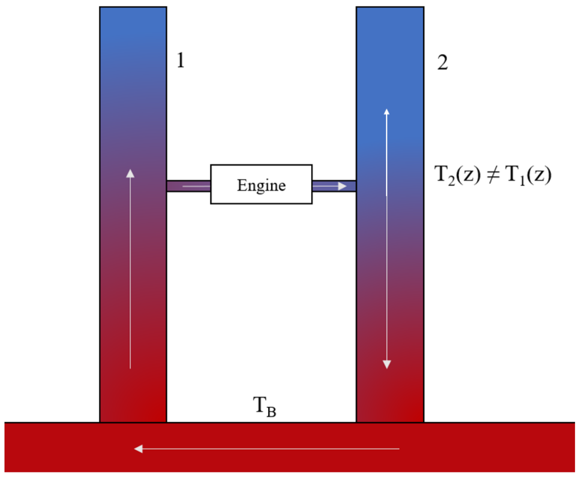

Figure 2: Inequality in temperature gradient creating a perpetual motion engine

We may now place a heat engine on the conducting rod at height z. The columns, under our assumption, act perpetually as thermal reservoirs at different temperatures—even while the system is nominally in equilibrium. This would allow us to extract work indefinitely, constructing a perpetual motion machine of the second kind—an absurdity that directly violates the second law of thermodynamics. Therefore, our initial assumption that $T_1(z) \neq T_2(z)$ must be false.

Maxwell reinforces this theoretical argument with an empirical observation: since we do not observe temperature gradients in columns of ideal gas at equilibrium, no substance ought to exhibit such gradients. If such a gradient existed, one could exploit the difference to violate Clausius's formulation of the second law (and, because Clausius' and Kelvin's formulations are logically equivalent, students are encouraged to reflect on how the same hypothetical apparatus would violate Kelvin's version as well).

3 (Modified) Santiago-Visser's Argument

I now present a modified version of a proof—originally due to Santiago and Visser [2]—demonstrating the existence of temperature gradients in a photon gas column. In their foundational work[4], Tolman and Ehrenfest showed that temperature at thermal equilibrium need not remain constant in curved spacetime, but instead varies with gravitational potential. Their derivation, however, relied on the machinery of general relativistic hydrodynamics—well beyond the scope of most undergraduate thermal physics curricula.

Santiago and Visser offered a more accessible approach, using only the concept of gravitational redshift to reach the same conclusion. Since redshift can be derived within the framework of special relativity, students already familiar with undergraduate electromagnetism should be able to grasp the argument with minimal additional background.

In adapting their proof, I depart slightly from the original treatment: instead of assuming a uniform gravitational field, I consider a spherically symmetric one. This choice streamlines the transition to the third argument in this paper, which addresses electromagnetic contributions to equilibrium gradients.

Let us now outline the setup. Consider a photon gas column situated within a spherically symmetric gravitational field, offset slightly from the radial direction (see Fig. 3). Suppose an observer located far from the column measures the spectral radiance of each segment and finds that the peak wavelength remains constant over time and position. By Wien's displacement law, the observer concludes that the system is in thermal equilibrium: the temperature of the column appears spatially and temporally uniform.

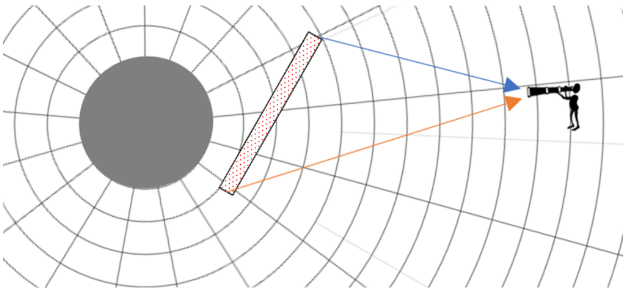

Figure 3: An observer observing the photons leaking from a photon gas column near a massive body

However, it is well known that as photons traverse a gravitational well, they lose energy and undergo a redshift in wavelength. In the case of a static, spherically symmetric gravitational field—that is, Schwarzschild geometry—the expression for gravitational redshift is well established and takes the form:

$$\frac{\lambda_\infty}{\lambda_e} = \left(1 - \frac{R_s}{r}\right)^{-1/2}$$

where λ_e is the wavelength at emission, λ_∞ is the wavelength observed at infinity, $R_s = 2GM/c^2$ is the Schwarzschild radius of the massive body and r is the radius at which the photon was initially emitted. For our observer situated a large distance away from the body, $\lambda_o = \lambda_\infty$ is given by -

$$\lambda_o = \lambda_e \left(1 - \frac{2GM}{c^2 r}\right)^{-1/2}$$

From Wien's Displacement Law, we know $\lambda_{o_{max}} T_o = \lambda_{e_{max}} T_e$. Therefore, the temperature recorded by the observer would be off by a factor of -

$$T_e = \frac{T_o}{\sqrt{1 - \frac{2GM}{c^2 r}}}$$

In our hypothetical scenario, however, different segments of the photon column reside at varying distances from the center of the massive body. As a result, photons originating from different heights should experience differing amounts of gravitational redshift. This variation would manifest in the observed blackbody spectra: rather than a uniform spectral distribution, the observer would detect intensity peaks shifted differently along the column.

This presents a clear contradiction. The assumption of thermal equilibrium implies a spatially constant temperature, yet the differential redshift demands otherwise. The only resolution is that the column must possess a temperature gradient—one that precisely compensates for the gravitational redshift. Only then would the observer perceive a consistent peak wavelength and, by extension, a constant temperature.

Thus, we are led to the conclusion that although the temperature of the photon gas is constant in time, it must vary with position. The locally measured temperature is, in equilibrium, a spatial function shaped by the geometry of the gravitational field.

$$T(r) = \frac{T_o}{\sqrt{1 - \frac{2GM}{c^2 r}}}$$

We were working in the Schwarzschild Geometry, the metric ($g_{\mu\nu}$) for which is

$$ds^2 = -\alpha c^2 dt^2 + \alpha^{-1} dr^2 + r^2 (d\theta^2 + \sin^2\theta d\phi^2)$$

where,

$$\alpha(r) = \left(1 - \frac{R_s}{r}\right)$$

On comparing with our expression for locally measured temperature, it becomes clear that the Temperature gradient of the photon gas column follows the expression -

$$T(r) = \frac{T_o}{\sqrt{-g_{tt}(r)}} \qquad (1)$$

where T_o is constant as described earlier (g_{tt} is the component of the metric tensor that serves as the coefficient of $c^2 dt^2$ term). It is important to note that the temperature gradient derived here is independent of time. This result is deeply counter-intuitive: although a spatial temperature gradient exists within the photon gas column, no heat flows from the hotter to the cooler regions. Thermal equilibrium is preserved—not through uniform temperature, but through a precise balance between thermal variation and spacetime curvature.

As is often the case in relativity, one must be attentive to the distinction between what is measured locally and what is defined globally. Just as notions of length and time differ between frames, so too must we distinguish between local temperature—measured by an observer comoving with the system—and coordinate temperature, which describes the system in a broader geometric frame. While not standard terminology, this distinction helps clarify why a temperature gradient does not, in this context, imply thermal disequilibrium.

3.1 Connecting the Two Pieces

Earlier, using Maxwell's argument, we established that the presence of unequal temperature gradients between two columns at equilibrium would enable the construction of a perpetual motion machine—an outcome forbidden by the second law of thermodynamics. Separately, we showed that a photon gas must exhibit a temperature gradient in a gravitational field in order to remain in thermal equilibrium with respect to an external observer. By connecting these two observations, we arrive at a general result: the temperature gradient described by Eq.1 must hold for all materials in static spacetimes—not just photon gases.

The proof follows the same logic as before. Suppose, hypothetically, that only the photon gas column exhibits a temperature gradient, while a second column—say, one composed of an ideal gas—maintains uniform temperature at equilibrium. Placing the two columns parallel and in close proximity (see Fig. 4), and thermally connecting their bases, we recreate the conditions described in Fig. 2. Once again, a horizontal conducting rod between the two at some height would permit continuous heat flow and indefinite work extraction—a direct violation of the second law.

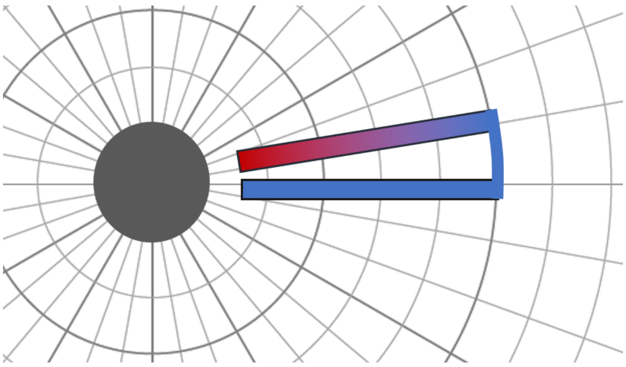

Figure 4: An observer observing the photons leaking from a photon gas column near a massive body

3.2 Universality of Gravity

To avoid the paradox of perpetual motion, we must conclude that all temperature gradients at thermal equilibrium within a given geometry must obey the same relation—namely, that described by Eq. 1. It is important to note, however, that this expression holds only in static spacetimes. The gradient itself arises from spacetime curvature: gravity alters the conditions of thermal equilibrium, and therefore any form of matter or radiation that couples to gravity must experience the same temperature gradient. In the non-relativistic limit $c \rightarrow \infty$ the gradient would be indeed in the limit $\nabla T(r) \rightarrow 0$. This convergence reaffirms what we may now call the universality of gravity—its unique role in shaping equilibrium without violating thermodynamic laws.

Having established gravity's universality and its influence on equilibrium temperature distributions, we now turn to a natural question: can other fields, such as electromagnetism, give rise to similar temperature gradients? This next argument, adapted from [2], builds again on Maxwell's two-column setup. We consider a similar apparatus as described earlier with a few minor adjustments. Suppose one of the columns is filled with very low density electron gas and the entire apparatus is subjected to an Electric Field $\vec{E}$ as in Fig. 5. Does $\vec{E}$ produce a temperature gradient at thermal equilibrium? Let us begin, as before, by assuming that it does—and follow the consequences.

If there is a temperature gradient produced due to the electric field then it must only affect those particles that interact with $\vec{E}$ to have any causal relationship in the first place. If the adjacent column is made of non-interacting particles (such as Neutron Gas) then $\vec{E}$ has no causal influence over the second column. We are thus led to an unsettling situation: one column (electron gas)

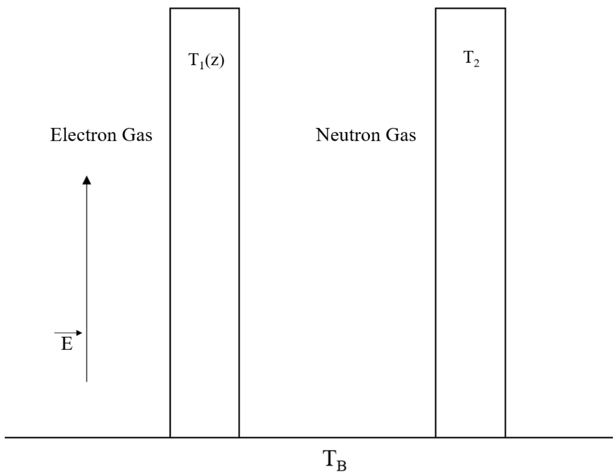

Figure 5: Electron Gas and Neutron Gas Columns Exposed to $\vec{E}$

exhibits a temperature gradient at thermal equilibrium, while the other (neutron gas) does not. Having now invoked Maxwell's argument twice, this should raise immediate concern. But to make the contradiction explicit: if, at any given height in the apparatus, the two columns maintain unequal temperatures while in thermal equilibrium, one could insert a heat engine between them and extract work ad infinitum. This would violate the second law of thermodynamics.

The lesson generalizes: any force that does not act universally cannot produce temperature gradients at thermal equilibrium. Gravity alone satisfies this condition—coupling to all forms of energy and matter—and thus gives rise to the Tolman–Ehrenfest effect. The electric field, by contrast, is selective in its coupling, and therefore cannot reshape thermal equilibrium in this way.

4 Influence of Electric Fields on Temperature Gradients

It is important to note that in the preceding argument, the influence of gravity was deliberately set aside in order to isolate the effect of the electric field. Let us now reintroduce gravity and ask: Does the presence of an electric field modify the temperature gradient at thermal equilibrium once gravitational effects are taken into account?

Santiago and Visser have argued that electric fields do not directly contribute to the development of temperature gradients. However, to probe this question more carefully, we must turn our attention to how electric fields and gravity are intertwined in relativistic physics. Specifically, let us recall two of Maxwell's equations.

$$\nabla.\vec{E} = \frac{\rho}{\epsilon_0} \text{ and } \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

To sustain an electric field, there must exist either a charge density or a time-varying magnetic field. In other words, electric field lines must either terminate on electric charges or form closed loops governed by Faraday's law.

4.1 Reissner–Nordström Geometry

In general relativity, any entity possessing energy and momentum contributes to the curvature of spacetime. This includes not only massive particles, but also fields—such as the electromagnetic field. The simplest setting in which to study the gravitational influence of a massive, charged object is the Reissner–Nordström metric: a static, spherically symmetric solution to the Einstein-Maxwell equations. The spacetime geometry around such an object—a charged, non-rotating black hole—is described by the following line metric element:

$$ds^2 = -\Delta c^2 dt^2 + \Delta^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

where the coefficient Δ is

$$\Delta(r) = \left(1 - \frac{R_s}{r} + \frac{R_Q^2}{r^2}\right)$$

Like earlier, $R_s = 2GM/c^2$ is the Schwarzschild radius and $R_Q = (Q^2 G)/(4\pi\epsilon_0 c^4)$ is a characteristic length defined by the net charge content of the body. Clearly, when we set $Q = 0$, we simply get a Schwarzschild geometry. If the black hole is also spinning, the geometry generalizes to the Kerr–Newman solution.

4.2 Are temperature gradients affected by $\vec{E}$?

We now return back to the original question - are temperature gradients at equilibrium (in static spacetime) affected by the presence of electric field? If the electric field could influence the temperature gradient at thermal equilibrium then it would allow for the existence of perpetual motion machines. Therefore, we rule out the possibility of a contribution by the electric field at thermal equilibrium. However, in static spacetimes, electric fields do not exist in isolation—they require a source, namely, electric charge. And the presence of charge, as we have seen, alters the spacetime geometry. Since the metric affects the temperature gradient, the presence of $\vec{E}$ does indeed contribute to the temperature gradient-indirectly, through its gravitational imprint on spacetime.

In routine thermodynamic contexts, the resulting temperature gradients are extraordinarily small—whether or not charge is present. For all practical purposes, they can be neglected. But for the sake of logical consistency—and for the coherence of thermodynamics in curved spacetime—the Tolman gradient must exist. It is a quiet but essential feature of any complete theory.

5 Conclusion

This paper proposed a pedagogical aid to help early undergraduate students engage with the limitations of the conventional formulations of thermodynamic laws. It also offered a conceptual clarification regarding the causal relationship between temperature gradients at thermal equilibrium and the presence of electric fields. Upon carefully analyzing the role of electric fields—while keeping the universality of gravity in view—it was shown that electric fields can influence temperature gradients, but only indirectly, through their effect on spacetime geometry. Since this influence is mediated by gravity itself, the conclusion remains consistent with the broader principle: gravity is the only force capable of producing temperature gradients at equilibrium.

Although the magnitudes of these gradients are negligible in routine experiments, their very existence requires a reconsideration of the foundational statements of thermodynamics. The zeroth law's definition of temperature is not compatible with relativity, and the second law's prescription for the direction of heat flow is challenged by the possibility of stable gradients in equilibrium. Fortunately, the field of relativistic thermodynamics is mature, and such foundational tensions have been addressed within its framework.

6 Discussion

Talks based on this work that were delivered to undergraduate physics audience received encouraging feedback, particularly in stimulating interest in relativistic thermodynamics and introducing concepts not typically covered in undergraduate thermal physics courses. While a structured assessment (such as a short quiz following a dedicated lecture) could offer insight into the accessibility and comprehension of these ideas at the undergraduate level, a detailed data-driven pedagogical analysis lies outside the scope of the present paper.

Nonetheless, any effective teaching module should invite both forward and backward modes of self-directed learning. A forward approach builds on results presented in class, encouraging students to apply newly acquired tools to extended problems. This might involve reformulating standard textbook exercises to include equilibrium temperature gradients, or exploring the role of relativistic thermodynamics in cosmology. A backward approach, by contrast, challenges foundational premises introduced without proof, prompting students to seek deeper theoretical grounding. This could involve studying Einstein's field equations to understand the gravitational role of the stress-energy tensor, or generalizing the Tolman result to stationary spacetime. The material presented here supports both trajectories, offering students an accessible yet conceptually rich path into the deeper structure of thermodynamics in curved spacetime.

Acknowledgments

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